Amplitude and Decibels

Sound is variation of air pressure and air pressure is measured in units of force per area. The unit of force is usually Newton (where one Newton is approximately the force with which a weight of $0.1kg$ is dragged down to the ground here on earth) and area is measured in m^2. The unit for pressure is therefore N/m^2 which is also referred to as 'Pascal', that is: $1\ N/m^2 = 1\ Pa$. As the interesting component of the air pressure (the sound component $p(t)$) is time-varying and therefore a signal, the following discussion will assume that we are dealing with the RMS-value of that signal and denote that simply with p - leaving out the time-dependence.

Human perception of loudness

The amplitude range of human hearing spans roughly from $0.00002\ Pa = 2 \cdot 10^{-5}\ Pa$ to $1500\ Pa = 1.5 \cdot 10^2\ Pa$ where the former is the threshold of hearing (the lowest amplitude which is required to hear anything at all) and the latter is the threshold of pain (where sound becomes so loud that it will be painful and potentially damaging to the ear). As it turns out, the perception of loudness is nonlinear. That means, when we have a sound pressure of $1\ Pa$ and increase that by $0.5\ Pa$ to yield $1.5\ Pa$, the perceived difference in loudness is larger than when we increase from $10\ Pa$ by $0.5\ Pa$ to $10.5\ Pa$. The intuitive interpretation is, that a difference of $0.5\ Pa$ in the former case is a greater fraction of the point of departure (namely 50%) where in the latter case it is only 5%. That suggests to measure the sound pressure difference not in absolute terms but always in relation to what is already there. That is, we want to have a perceptive measure, for which an increase from 1.0 to 1.5 leads to the same measured difference as an increase from 10 to 15. Lets identify a reference sound-pressure (our point of departure) with p and a perturbation from that reference sound-pressure by Δp. The key is to express the perturbation in terms of a fraction of the reference value - simply by dividing it by the reference value: $\frac{\Delta p}{p}$. Furthermore, let's identify the perceived loudness due to some sound-pressure p by L_p and the perceived difference due to some sound-pressure difference Δp by ΔL_p. The perceived difference ΔL_p should be proportional to the physical difference Δp expressed as fraction of the referenvce value p. Mathematically:

$$\Delta L_p = c \cdot \frac{\Delta p}{p} \tag{1}$$

for some constant c. By applying some math (replacing the differences with differentials and integrating both sides), we obtain the desired relationship between L_p and p:

$$L_p = c \cdot \ln(p) \tag{2}$$

where ln is the natural logarithm. That equation says in words: our **perception of loudness is proportional to the logarithm of the sound-pressure**.

Various Definitions of the Decibel

The definition of the decibel (dB) can be motivated from the observation in equation 2. For a logarithmic measure such as the decibel, we have to choose some reference value, express the measurement values as proportion of this reference value (by dividing by it) and then taking the logarithm. Because $\log(1) = 0$, we will get a dB-value of 0 when our measurement variables in equal to the reference value, positive values, when it's above and negative values when it's below.

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Decibels in Acoustics: The Sound Pressure Level SPL

For the definition of the SPL measure, the threshold of hearing was chosen as reference value and the logarithm to base 10 is used instead of natural logarithm and the result is finally scaled by the constant 20:

$$L_p = 20 \cdot \log\left(\frac{p}{p_0}\right) \tag{3}$$

These adjustments make no difference conceptually, they just introduce some overall scaling for the numerical values. These adjustments were made, such that a level of 0 dB corresponds to the threshold of hearing (as a result of the division of p by p_0 and that an increase of 1 dB corresponds (roughly) to a just notable difference in perception (as a result of scaling by factor 20). Furthermore, as a convenient rule of thumb, with this definition we can say that a **doubling of the physical sound pressure leads to an increase of** $6.02dB$ **in the sound pressure level** and conversely, halving the RMS-value of the physical sound-pressure leads to a decrease of $6.02dB$. Note that for sound-pressures below p_0, the sound-pressure-level in dB will take on negative values.

Decibels in Electrical Engineering: dBu and dBV

In the studio, we usually do not only work with air pressure levels - these are relevant only at the physical front-end of the signal chain - namely the microphone. A microphone converts the sound-pressure into an electrical voltage which is proportional to the sound pressure in front of the microphone. More precisely, it should be proportional, but in practice it is not trivial to build a device for which this rule is strictly true and independent of frequency - but we will leave those details to the microphone-manufacturers and assume that they did a good job. So now we have to deal with an electrical voltage, for which we also want to have a logarithmic measure expressed in decibels. Let's call that voltage level L_V, then we have:

$$L_V = 20 \cdot \log\left(\frac{V}{V_0}\right) \tag{4}$$

where - again - some reference value V_0 must be chosen. Different conventions exists for the reference voltage V_0. The most important ones are: $V_0 = 1\,Volt$ in which case the unit is sometimes called dBV or $V_0 = 0.775\,Volt$ in which case the unit is called dBu.

Decibels in Digital Audio: $dBFS$

Later in the signal chain, we usually convert the electrical voltage into a digital representation which are merely numbers without any physical unit. In a typical DAW (**D**igital **A**udio **W**orkstation) environment, these numbers are represented as floating point numbers (with 32 or 64 bits). For the definition of a dB-level on such a digital representation, we do not have to deal with physical units anymore, therefore we denote a generic signal amplitude with A (which can now stand for a voltage, sound-pressure or whatever) and we denote the reference amplitude as A_0, such that the definition of the level becomes:

$$L = 20 \cdot \log\left(\frac{A}{A_0}\right) \tag{5}$$

For floating point representations such as they occur inside a DAW, the reference amplitude A_0 is chosen to be unity. As long as we stick to the floating point format, we can easily deal with numbers that are

much smaller and also much larger than unity. However, ultimately we must convert the number into a fixed point representation because the digital/analog-converter (DAC) needs the signal in this form. For fixed point numbers, the reference amplitude A_0 is often chosen to be the threshold of clipping, that is: largest amplitude which can be represented by the digital number system used. Therefore the dB measure in this representation is also called $dBFS$ for dB full scale. In the conversion-process from float to fixed, we normally map signals at unit amplitude to the full scale. That's why an amplitude of 1 is usually the maximum amplitude we want to have inside the DAW and in practice we mostly deal with signal amplitudes below 1. And because 1 was chosen to map to 0 dB, typical signal levels in dB will be negative.

Decibels all over the place: $dB...foo$

I have listed the dB-definitions which are probably the most important ones for the audio-engineer, but it should be mentioned that there are a whole bunch of other dB definitions as well (each of which with its own suffix), so always watch out. In the noise control domain, for example, often units like $dB(A), dB(B), dB(C)$ occur - these are sound pressure levels but with certain frequency weightings to take into account different sensitivities of the human hearing system to different frequencies.